Exercise 160

For the following exercises, solve the trigonometric equations on the interval $0 \leq \theta < 2\pi$.

$$3 \sec \theta - 2\sqrt{3} = 0$$
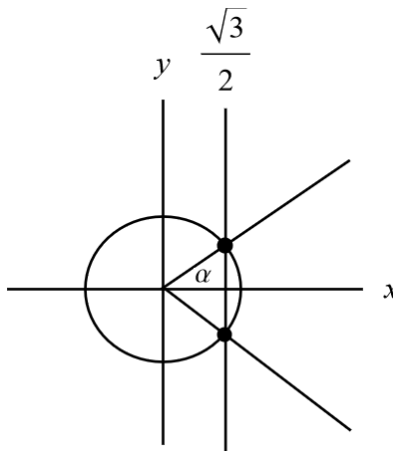
Solution

$$3 \sec \theta - 2\sqrt{3} = 0$$

$$3 \sec \theta = 2\sqrt{3}$$

$$\sec \theta = \frac{2}{\sqrt{3}}$$

$$\frac{1}{\cos \theta} = \frac{2}{\sqrt{3}}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

The aim is to find the angles to the two points on the unit circle that are $\sqrt{3}/2$ units to the right.

Taking the inverse cosine of $\sqrt{3}/2$ gives 30°, or $\pi/6$ radians. This is α in the figure.

$$\alpha = \frac{\pi}{6}$$

The angle to the point below it is the same but negative, $-\pi/6$. Since every angle has to be between 0 and 2π, add 2π to it.

$$-\frac{\pi}{6} + 2\pi = \frac{11\pi}{6}$$

Therefore,

$$\theta = \left\{ \frac{\pi}{6}, \frac{11\pi}{6} \right\}.$$